Name: Date: Period:

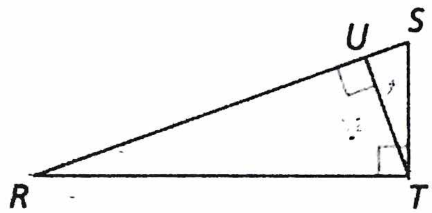
Similar Right Triangles

Lesson Objective IWBAT identify and use ~ right Δs and find geometric means

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

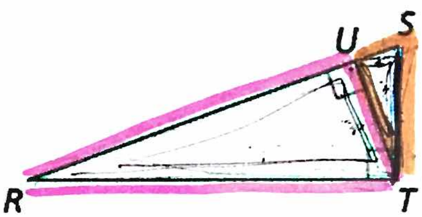
Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

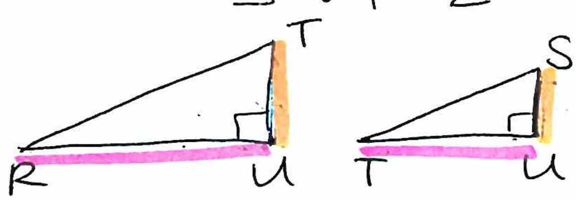


Example:

1. Identify the similar triangles in the diagram, and write a similarity statement.

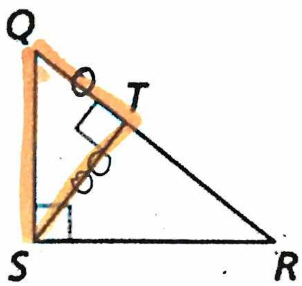


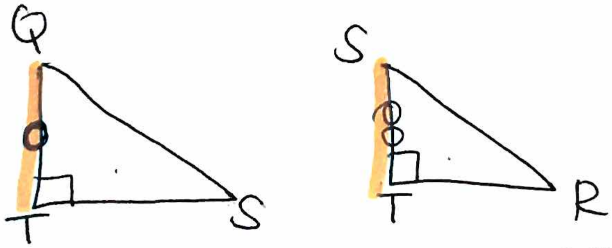
$\Delta SUT \sim \Delta$ TUR $\sim \Delta$ STR



$\Delta RTS \sim \Delta RUT \sim \Delta TUS$

2. Identify the similar triangles in the diagram, and write a similarity statement.

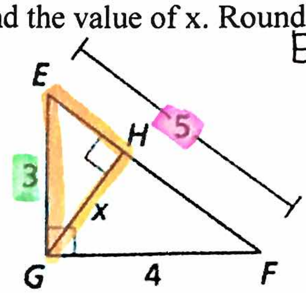


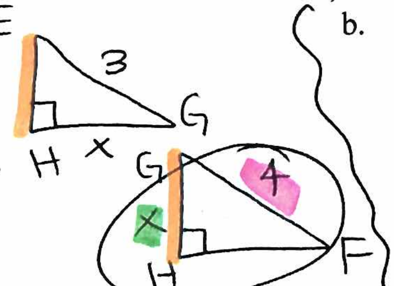


$\Delta SQT \sim \Delta$ RST $\sim \Delta$ RQS

3. Find the value of x. Round to the nearest hundredth, if necessary.

a.

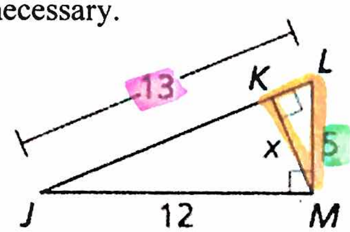


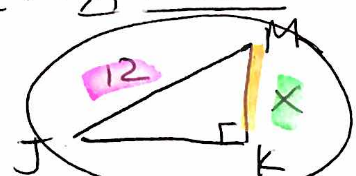


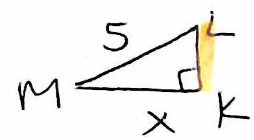
$\frac{4}{5} = \frac{x}{3} \rightarrow 12 = 5x$

$\boxed{x = 2.4}$

b.



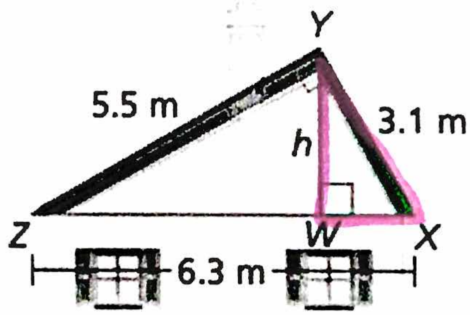


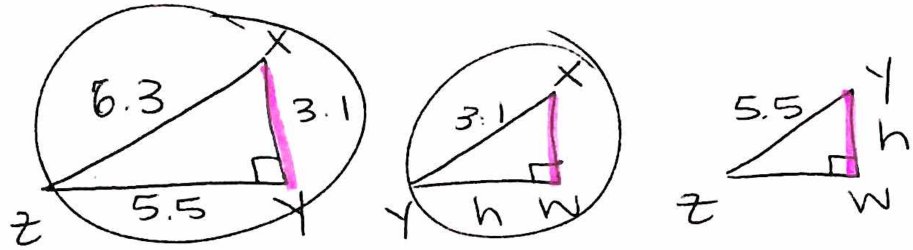


$\frac{12}{13} = \frac{x}{5}$

$60 = 13x \rightarrow \boxed{x \approx 4.62}$

4. A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the roof. Round to the nearest hundredth, if necessary.





$$\frac{3.1}{6.3} = \frac{h}{5.5} \rightarrow \boxed{h \approx 2.71 \text{ m}}$$

Geometric Mean: The geometric mean of two positive integers a and b is the positive number x that satisfies

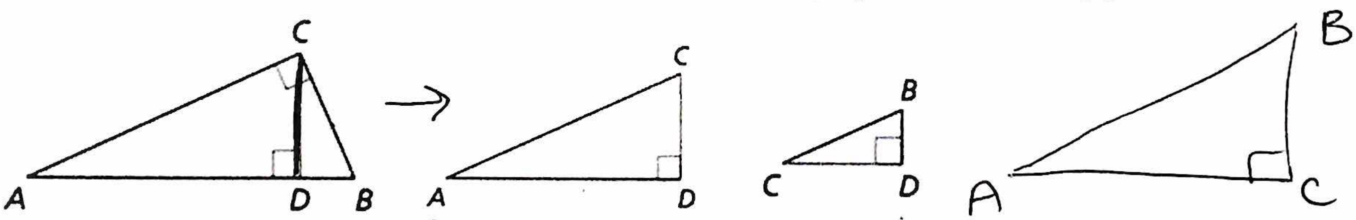
$\frac{a}{x} = \frac{x}{b}$. So... $\rightarrow \sqrt{x^2} = \sqrt{a \cdot b} \rightarrow x = \pm\sqrt{a \cdot b} \rightarrow x = \sqrt{a \cdot b}$

Example:

5. Find the geometric mean of 24 and 48. Find the EXACT value, and round to the nearest hundredth, if necessary.

$$x = \sqrt{24 \cdot 48}$$
$$x = \sqrt{4 \cdot 6 \cdot 4 \cdot 12}$$
$$x = \sqrt{4 \cdot 6 \cdot 4 \cdot 6 \cdot 2}$$
$$x = 4 \cdot 6 \cdot \sqrt{2}$$
$$\boxed{x = 24\sqrt{2}}$$
$$\boxed{x \approx 33.94}$$

In right ΔABC, altitude $\overline{CD}$ is drawn to the hypotenuse, forming two smaller right triangles that are similar to ΔABC. From the Right Triangle Similarity Theorem, you know that $\Delta CBD \sim \Delta ACD \sim \Delta ABC$. Because the triangles are similar, you can write and simplify the following proportions involving geometric means.



$$\frac{AD}{CD} = \frac{CD}{BD} \qquad \frac{BD}{BC} = \frac{BC}{AB} \qquad \frac{AD}{AC} = \frac{AC}{AB}$$

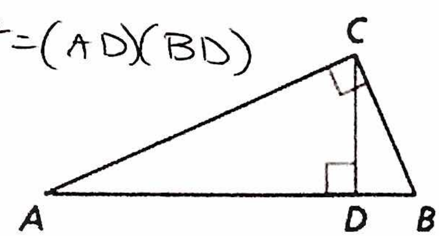
$$CD^2 = (AD)(BD) \qquad BC^2 = (AB)(BD) \qquad AC^2 = (AD)(AB)$$

Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.

$CD^2 = (AD)(BD)$



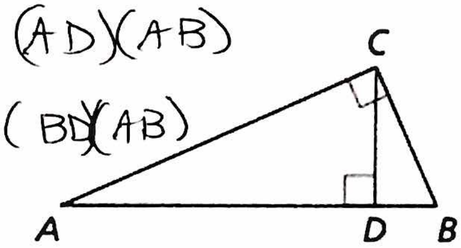
Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

$AC^2 = (AD)(AB)$

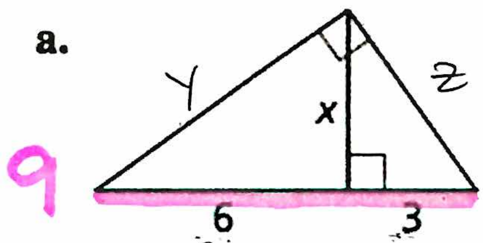
$BC^2 = (BD)(AB)$



Examples:

6. Find the value of each variable.

a.



$z^2=(9)(3)$

$z=\sqrt{9\cdot 3}$

$\boxed{z=3\sqrt{3}}$

$x^2=(6)(3)$

$x=\sqrt{18}$

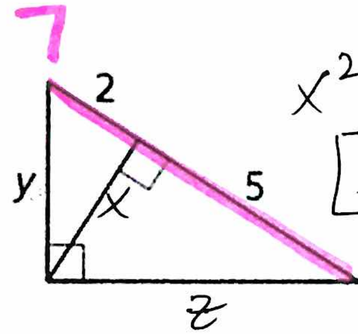
$\boxed{x=3\sqrt{2}}$

$y^2=(9)(6)$

$y=\sqrt{9\cdot 6}$

$\boxed{y=3\sqrt{6}}$

b.



$x^2=(2)(5)$

$\boxed{x=\sqrt{10}}$

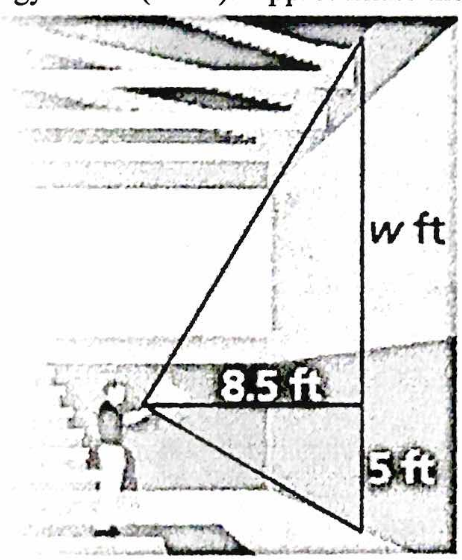
$y^2=(7)(2)$

$\boxed{y=\sqrt{14}}$

$z^2=(7)(5)$

$\boxed{z=\sqrt{35}}$

7. To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall. You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye (5 ft) and the horizontal distance from you to the gym wall (8.5 ft). Approximate the height of the gym wall. Round to the nearest hundredth.



$8.5^2=(w)(5)$

$5w=72.25$

$\boxed{w=14.45 \text{ ft}}$

$+5 \text{ ft}$

gym wall = 19.45 ft tall